The Model Matrix

Lecture 8

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Outline

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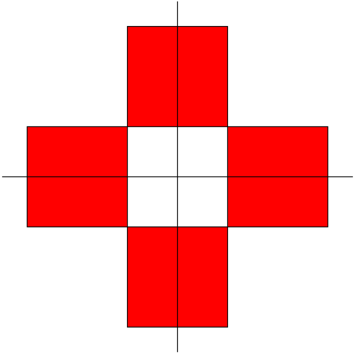
The Model Coordinate System

- When we create an object, such as a square or a circle, we use the coordinate system and position that are most convenient.
  - To create a square, we might place the lower-left corner at $(0, 0)$ and let the side be 1.
  - To create a circle, we would place the center at $(0, 0)$ and let the radius be 1.
- That coordinate system is call the model coordinate system and it is specific to each object.

The Model Coordinate System



- For example, suppose that we want to draw four squares as shown.

The Model Coordinate System

- Should we construct 4 separate squares in four separate buffers?

The Model Coordinate System

- Should we construct 4 separate squares in four separate buffers?
- Or should we construct one square and draw it 4 times, in 4 different locations?

The Model Coordinate System

- Should we construct 4 separate squares in four separate buffers?
- Or should we construct one square and draw it 4 times, in 4 different locations?
- How do we change the location (in world coordinates) of an object?

Outline

The Model Matrix

- The model matrix is a matrix that represents a geometric transformation that will move or modify (i.e., transform) an object from its model coordinates to world coordinates.
- The model matrix consists of any combination of
  - Translations – slide in a given direction.
  - Rotations – rotate about a given axis.
  - Scalings – stretch or shrink by a given factor.
  - Or any other transformation that can be represented by a matrix.

The Model Matrix

The Model Matrix

```
mat4 model = ··· // Create the (global) 4 x 4 model matrix

GLuint model_loc = glGetUniformLocation(program, "model");

glUniformMatrix4fv(model_loc, 1, GL_FALSE, model);
```

- The model matrix, like the projection matrix, must be passed to the vertex shader.
- The vertex shader will apply it, along with the projection matrix, to the vertex.

The Vertex Shader

The Vertex Shader

```
#version 450 core

uniform mat4 model;
uniform mat4 proj;

out vec4 color;

layout (location = 0) in vec2 vPosition;
layout (location = 1) in vec3 vColor;

void main()
{
    gl_Position = proj*model*vec4(vPosition, 0.0f, 1.0f);
    color = vec4(vColor, 1.0f);
}
```

Outline

Translations

Translations

```
mat4 translate(float dx, float dy, float dz);
```

- The `translate()` function will return a translation matrix.
- The x, y, and z coordinates will be shifted by the amounts `dx`, `dy`, and `dz`, respectively.
- See `vmath.h` for details.

Translations

Translation Matrix

$$\mathbf{T} = \begin{pmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

- Translates $(x, y, z, 1)$ to $(x + d_x, y + d_y, z + d_z, 1)$.

Translations

Translation

$$\begin{pmatrix} x+d_x \\ y+d_y \\ z+d_z \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}.$$

- Translates $(x, y, z, 1)$ to $(x + d_x, y + d_y, z + d_z, 1)$.

Outline

Rotations

Rotations

```
mat4 rotate(float angle, float ax, float ay, float az);
```

- The `rotate()` function will return a rotation matrix.
- The object will be rotated through the given angle and about an axis through the origin and the given point (`ax`, `ay`, `az`).
- The direction of rotation is determined by the right-hand rule: point your right thumb in the direction from the origin to the point and curl your fingers.
- See `vmath.h` for details.

Rotations About the z-Axis

Rotation Matrix

$$\mathbf{R_z} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

- Rotation about the z-axis.
- Rotates $(x, y, z, 1)$ to $(x\cos\theta - y\sin\theta, x\sin\theta + y\cos\theta, z, 1)$.

Rotations About the z-Axis

Rotation

$$\begin{pmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

- Rotation about the z-axis.
- Rotates $(x, y, z, 1)$ to $(x\cos\theta - y\sin\theta, x\sin\theta + y\cos\theta, z, 1)$.

Outline

Scalings

Scalings

```
mat4 scale(float sx, float sy, float sz);
```

- The `scale()` function will return a scaling matrix.
- The object will be stretched or shrunk by factors `sx`, `sy`, and `sz` in the *x*, *y*, and *z* directions, respectively.
- If one of the values is -1 and the other two are 1, then the scaling will be a reflection.
- None of `sx`, `sy`, and `sz` should ever be `0`.
- See `vmath.h` for details.

Scalings

Scaling Matrix

$$\mathbf{S} = \begin{pmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

- Scales $(x, y, z, 1)$ to $(s_x \cdot x, s_y \cdot y, s_z \cdot z, 1)$.

Scalings

Scaling Matrix

$$\begin{pmatrix} s_x \cdot x \\ s_y \cdot y \\ s_z \cdot z \\ 1 \end{pmatrix} = \begin{pmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

- Scales $(x, y, z, 1)$ to $(s_x \cdot x, s_y \cdot y, s_z \cdot z, 1)$.

Outline

Sequences of Transformations

- In most cases, an object will go through a sequence of transformations.
- All sequences of transformations can be consolidated down to
  - A scaling, followed by
  - A rotation, followed by
  - A translation.
- This is the most intuitive sequence.

Sequences of Transformations

- A translation followed by a rotation can be rewritten as a rotation followed by a translation.
- A translation followed by a scaling can be rewritten as a scaling followed by a translation.
- A rotation followed by a scaling can be rewritten as a scaling followed by a rotation.

Sequences of Transformations

- Furthermore, the product of two translations is again a translation.
- The product of two rotations is again a rotation.
- The product of two scalings is again a scaling.
- Thus, any sequence can be rewritten as one scaling, then one rotation, then one translation.

Outline

Generating Transformation Matrices

Generating Transformation Matrices

```
mat4 translate(dx, dy, dz);
mat4 rotate(angle, ax, ay, az);
mat4 scale(sx, sy, sz);
```

- The `vmath.h` library has functions that will generate transformation matrices for translations, rotations, and scalings.
- Each matrix is returned in *column-major* order.

Outline

Other Rotations

- What if we want to rotate about a point (x_0, y_0) that is not the origin?
- We can translate (x_0, y_0) to the origin, rotate, then translate the origin back to (x_0, y_0).

Other Rotations

Other Rotations

```
model = translate(-x_0, -y_0, 0.0f)*model;
model = rotate(angle, 0.0f, 0.0f, 1.0f)*model;
model = translate(x_0, y_0, 0.0f)*model;
```

Other Rotations

```
model = translate(x_0, y_0, 0.0f)
          *rotate(angle, 0.0f, 0.0f, 1.0f)
            *translate(-x_0, -y_0, 0.0f)*model;
```

Other Scalings

- What if we want to scale about a fixed point (x_0, y_0) that is not the origin?
- We can translate (x_0, y_0) to the origin, scale, then translate the origin back to (x_0, y_0).

Other Scalings

Other Scalings

```
model = translate(-x_0, -y_0, 0.0f)*model;
model = scale(s_x, s_y, s_z)*model;
model = translate(x_0, y_0, 0.0f)*model;
```

Other Scalings

```
model = translate(x_0, y_0, 0.0f)
          *scale(s_x, s_y, s_z)
            *translate(-x_0, -y_0, 0.0f)*model;
```

Outline

Assignment

Assignment

- Assignment 7.
- Read pp. 207 - 210, Homogeneous Coordinates.
- Read pp. 210 - 217, Linear Transformations and Matrices.